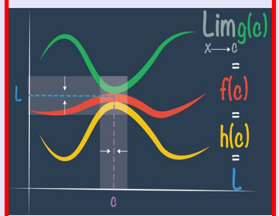
Calculus I

Lecture 11



Feb 19-8:47 AM

Class Quiz 4

Evaluate $\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} = \frac{\sqrt{1+0}-1}{0} = \frac{1-1}{0} = \frac{0}{0}$ I.F.

$$\lim_{h\to 0} \frac{(\overset{A}{\sqrt{1+h}} \overset{-B}{-1})(\overset{A}{\sqrt{1+h}} \overset{+B}{+1})}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{\overset{A^2-B^2}{\cancel{1}+h-\cancel{1}}}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1} = \frac{1}{\sqrt{1+0}+1}$$

$$= \boxed{\frac{1}{2}}$$

Sep 12-7:17 AM

Prove $\lim_{x \to 2} \left(\frac{1}{2}x - 1\right) = 0$

$f(x) = \frac{1}{2}x - 1$

$a = 2$

$L = 0$ ✓

For any $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|\frac{1}{2}x - 1 - 0\right| < \varepsilon$ whenever $|x - 2| < \delta$

$\left|\frac{1}{2}x - 1\right| < \varepsilon$ " $|x - 2| < \delta$

Multiply by 2

$|x - 2| < 2\varepsilon$

Pick $\delta = 2\varepsilon$

Sep 12-7:37 AM

Prove $\lim_{x \to 6} \left(-\frac{2}{3}x + 4\right) = 0$

$f(x) = -\frac{2}{3}x + 4$

$a = 6$

$L = 0$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$\left|-\frac{2}{3}x + 4 - 0\right| < \varepsilon$ whenever $|x - 6| < \delta$

multiply by 3

$|-2x + 12| < 3\varepsilon$

$|-2(x - 6)| < 3\varepsilon$

$|-2|\,|x - 6| < 3\varepsilon$

$2|x - 6| < 3\varepsilon$

$|x - 6| < \frac{3\varepsilon}{2}$

Pick $\delta = \frac{3\varepsilon}{2}$

If $\varepsilon = 2 \to \delta = 3$

If $\varepsilon = 1 \to \delta = \frac{3}{2}$

$\left|-\frac{2}{3}x + 4\right| < \varepsilon$

$\frac{3}{2}\left|-\frac{2}{3}x + 4\right| < \frac{3\varepsilon}{2}$

$|-x + 6| < \frac{3\varepsilon}{2}$

$|-(x - 6)| < \frac{3\varepsilon}{2}$

$|x - 6| < \frac{3\varepsilon}{2}$

Sep 12-7:42 AM

Prove $\lim_{x\to 1}(x^2+8x-9)=0$

$f(x)=x^2+8x-9$
$a=1$
$L=0$ ✓

For every $\varepsilon>0$, there is a $\delta>0$ such that $|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2+8x-9-0|<\varepsilon$ whenever $|x-1|<\delta$

$|x^2+8x-9|<\varepsilon$ " $|x-1|<\delta$

$|(x+9)(x-1)|<\varepsilon$

$|x+9|\,|x-1|<\varepsilon$

Bound Keep

When working with polynomial functions, we wish $\delta\le 1$

$|x-1|<1$

$-1<x-1<1$

Add 10

$-1+10<x-1+10<1+10$

$9<x+9<11$

$|x+9|<11$

$|x+9|\,|x-1|<11|x-1|<\varepsilon$

$|x-1|<\frac{\varepsilon}{11}$

Pick $\delta=\min\{1,\frac{\varepsilon}{11}\}$

If $\varepsilon=1 \to \delta=\frac{1}{11}$

If $\varepsilon=2 \to \delta=\frac{2}{11}$

If $\varepsilon=12 \to \delta=\min\{1,\frac{12}{11}\}=1$

Sep 12-7:56 AM

Prove $\lim_{x\to -2}(x^2-3x)=10$

$f(x)=x^2-3x$
$a=-2$
$L=10$ ✓

For every $\varepsilon>0$, there is a $\delta>0$ such that $|f(x)-L|<\varepsilon$ whenever $|x-a|<\delta$

$|x^2-3x-10|<\varepsilon$ whenever $|x-(-2)|<\delta$

$|(x-5)(x+2)|<\varepsilon$ whenever $|x+2|<\delta$

$|x-5|\,|x+2|<\varepsilon$

Bound Keep

For polynomial functions we wish $\delta\le 1$

$|x+2|<1$

$-1<x+2<1$

Subtract 7

$-1-7<x+2-7<1-7$

$-8<x-5<-6<8$

$|x-5|<8$

$|x-5|\,|x+2|<8|x+2|<\varepsilon$

$|x+2|<\frac{\varepsilon}{8}$

Pick $\delta=\min\{1,\frac{\varepsilon}{8}\}$

Sep 12-8:06 AM

Prove $\lim_{x \to 0} \sqrt[5]{x} = 0$

$f(x) = \sqrt[5]{x}$

$a = 0$

$L = 0$ ✓

For every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|\sqrt[5]{x} - 0| < \varepsilon$ whenever $|x - 0| < \delta$

$|\sqrt[5]{x}| < \varepsilon$ 〃 $|x| < \delta$

Raise both sides to the 5th Power

$(|\sqrt[5]{x}|)^5 < \varepsilon^5 \rightarrow |x| < \varepsilon^5$

Pick $\delta = \varepsilon^5$

Sep 12-8:17 AM

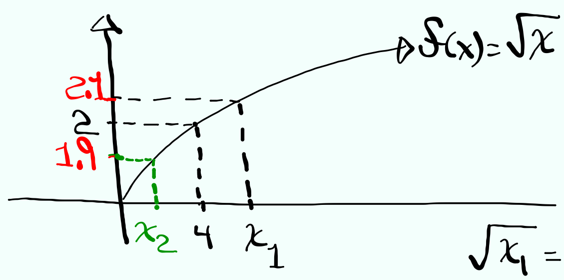
Use the graph below to find a $\delta > 0$ for the given ε.

$f(x) = \sqrt{x}$

$a = 4$

$L = 2$

$\varepsilon = .1$



$\sqrt{x_1} = 2.1$

$x_1 = (2.1)^2 = 4.41$

$\sqrt{x_2} = 1.9$

$x_2 = 1.9^2 = 3.61$

.39 .41

3.61 4 4.41

$\delta = \min\{.39, .41\} = \boxed{.39}$

Sep 12-8:23 AM

Prove $\lim_{x \to \frac{1}{2}} \frac{1}{x} = 2$

$f(x) = \frac{1}{x}$
$a = 1/2$
$L = 2$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

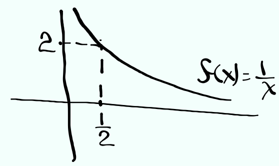
$\left| \frac{1}{x} - 2 \right| < \varepsilon$ whenever $\left| x - \frac{1}{2} \right| < \delta$

$\left| \frac{1 - 2x}{x} \right| < \varepsilon$

$\left| \frac{-2(x - \frac{1}{2})}{x} \right| < \varepsilon$

$\frac{2}{|x|} \left| x - \frac{1}{2} \right| < \varepsilon$

Bound Keep



If $\delta \le \frac{1}{4}$ - - -

Sep 12-8:29 AM